

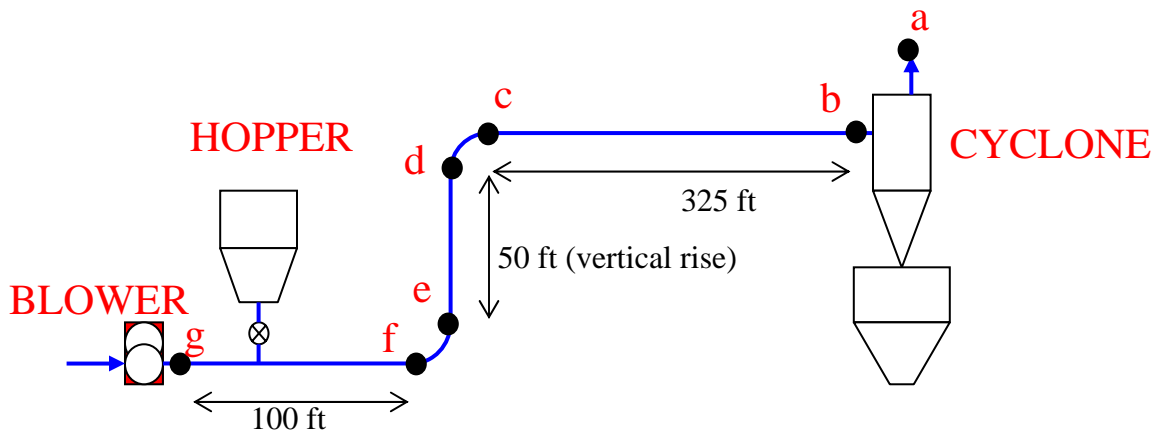
DESIGN EXAMPLE – DILUTE PHASE PNEUMATIC CONVEYING

A plastics production plant wants to increase the capacity through an existing conveying system. The existing system has 6 inch ID pipes and is configured as shown in the diagram below.

The High Density Polyethylene (HDPE) particles have an average size of 4 mm. The conveying gas is at 68°F. The existing blower can produce 1375 SCFM.

The desired capacity increase is from 20,000 lbm/hr to 30,000 lbm/hr. Can the existing blower and pipe system meet this increase in capacity?

Assume the pressure drop across the cyclone is 5 inches of water. The pressure drop across the blower inlet pipe and silencers is 0.3 psi. The pipe bends have $R/D = 6$. Pipe roughness is $k = 0.00015$ ft. The particles have density $\rho_p = 59$ lbm/ft³. Terminal velocity of the particles is $U_t = 30.6$ ft/s.



SOLUTION

Push conveying systems are conveniently calculated from exit to inlet (the exit pressure boundary condition is known). Hence, on the figure the points at which the pipe line is segmented are labeled 'a' through 'g' from exit to inlet.

a-b Cyclone

$P_a = 14.7$ psia (exit atmospheric pressure)

$P_b - P_a = \Delta P_{cyclone} = 5$ inches water (given)

$$P_b = P_a + \Delta P_{cyclone} = \underline{\hspace{2cm}}$$

b-c Horizontal Pipe

$L = 325$ ft

Use density and velocity values at point 'b'

$$\Delta P_{horiz} = (4f + \lambda_z \mu) \frac{L}{D} \frac{\rho_g V_g^2}{2g_c}$$

$$V_g = \frac{Q_{STP}}{A} \frac{P_{STP}}{P_b} = \frac{1375 \text{ ft}^3 / \text{min}}{0.1963 \text{ ft}^2} \frac{14.7 \text{ psi}}{P_b} \frac{\text{min}}{60s} = \underline{\hspace{2cm}}$$

Gas friction factor, from Churchill's equation

$$f = 2 \left[\left(\frac{8}{\text{Re}} \right)^{12} + \frac{1}{(A+B)^{3/2}} \right]^{1/12}$$

$$A = \left[2.457 \ln \left(\frac{1}{\left(\frac{7}{\text{Re}} \right)^{0.9} + 0.27 \frac{k}{D}} \right) \right]^{16}$$

$$B = \left[\frac{37530}{\text{Re}} \right]^{16}$$

$k = 0.00015$ ft, $\text{Re} = 363,037$

$$f = 0.00419$$

Gas density at point 'a' from ideal gas law

$$\rho_g = \frac{PM}{RT} = \frac{14.7 \text{ psi}}{10.73 \frac{\text{psi ft}^3}{\text{lbmol } ^\circ\text{R}}} \frac{29 \frac{\text{lbm}}{\text{lbmol}}}{528^\circ\text{R}}$$

$$= 0.0753 \text{ lbm / ft}^3$$

$$5'' \text{ water} \left(\frac{1 \text{ psi}}{27.68 \text{ inches}} \right) = 0.18 \text{ psi}$$

$P_b = 0.18 + 14.7 = 14.88$ psi

$$A = \frac{\pi D^2}{4} = \frac{\pi (0.5 \text{ ft})^2}{4} = 0.1963 \text{ ft}^2$$

Velocity at point 'b'

$$V_g = \frac{1375}{0.1963} \frac{14.7}{14.88} = 6920 \text{ ft / min}$$

$$= 115.3 \text{ ft / s}$$

Viscosity is insensitive to pressure
At 68°F

$$\mu_g = 0.017 \text{ cP} = 0.0000114 \frac{\text{lbm}}{\text{ft s}}$$

Gas density at point 'b'

$$\rho_g = \rho_{gSTP} \frac{P_b}{P_{STP}} = 0.0753 \frac{14.88}{4.7}$$

$$= 0.0762 \text{ lbm / ft}^3$$

$$\text{Re} = \frac{0.0762 \text{ lbm / ft}^3 \cdot 115.3 \text{ ft / s} \cdot 0.5 \text{ ft}}{0.000014 \text{ lbm / ft s}}$$

$$= 363,037$$

Solids mass to air mass ratio

$$\mu = \frac{w_s}{\rho_{gSTP} Q_{STP}} = \frac{30000 \text{ lbm/hr}}{0.0752 \text{ lbm/ft}^3 \cdot 1375 \text{ ft}^3/\text{min} \cdot 60 \text{ min/hr}}$$

$$= \underline{\hspace{2cm}}$$

$$\mu = 4.833$$

λ_z for particles with $d_p > 500$ microns

$$\lambda_z = 0.082 \mu^{-0.3} Fr^{-0.86} Fr_s^{0.25} \left(\frac{D}{d_p} \right)^{0.1}$$

$$d_p = 4 \text{ mm} = 0.01312 \text{ ft}$$

$$D = 0.5 \text{ ft}$$

$$Fr = \frac{(115.3 \text{ ft/s})^2}{32.174 \text{ ft/s}^2 \cdot 0.5 \text{ ft}} = 826.3$$

$$Fr_s = \frac{(30.6 \text{ ft/s})^2}{32.174 \text{ ft/s}^2 \cdot 0.5 \text{ ft}} = 2218$$

$$Fr = \frac{V_g^2}{gD} = \underline{\hspace{2cm}}$$

$$Fr_s = \frac{U_t^2}{gD} = \underline{\hspace{2cm}}$$

$$\lambda_z = 0.001564$$

$$\lambda_z = \underline{\hspace{2cm}}$$

Hence

$$\Delta P_{bc} = \underline{\hspace{2cm}}$$

$$\Delta P_{bc} = (4 \cdot 0.00419 + 0.001564 \cdot 4.833)$$

$$\times \frac{325 \cdot 0.0762 \text{ lbm/ft}^3 \cdot (115.3 \text{ ft/s})^2}{0.5 \cdot 2 \cdot 32.774 \frac{\text{lbm} \cdot \text{ft}}{\text{lb} \cdot \text{s}^2}}$$

$$\times \frac{\text{ft}^2}{144 \text{ in}^2}$$

$$= 1.729 \text{ psi}$$

At point 'c'

$$P_c = P_b + \Delta P_{bc} = \underline{\hspace{2cm}}$$

At point 'c'

$$P_c = 14.88 + 1.729 = 16.61 \text{ psi}$$

$$V_g = \frac{Q_{STP}}{A} \frac{P_{STP}}{P_c} = \frac{1375 \text{ ft}^3/\text{min} \cdot 14.7 \text{ psi} \cdot \text{min}}{0.1963 \text{ ft}^2 \cdot P_c \cdot 60 \text{ s}} = \underline{\hspace{2cm}}$$

$$V_g = 101.3 \text{ ft/s}$$

$$\rho_g = \rho_{gSTP} \frac{P_c}{P_{STP}} = 0.0753 \frac{P_c}{14.7} = \underline{\hspace{2cm}}$$

$$\rho_g = 0.0850 \text{ lbm/ft}^3$$

c-d Pipe Bend

$$\Delta P_{cd} = \underline{\hspace{2cm}}$$

$$\Delta P_{bend} = B(1 + \mu) \frac{\rho_g V_g^2}{2g_c}$$

$$\text{For } \frac{R}{D} > 6 \quad B = \underline{\hspace{2cm}}$$

$$\Delta P_{cd} = \underline{\hspace{2cm}}$$

At point 'd'

$$P_d = \underline{\hspace{2cm}}$$

$$V_g = \frac{Q_{STP}}{A} \frac{P_{STP}}{P_d} = \frac{1375 \text{ ft}^3 / \text{min}}{0.1963 \text{ ft}^2} \frac{14.7 \text{ psi}}{P_d} \frac{\text{min}}{60s} = \underline{\hspace{2cm}}$$

$$\rho_g = \rho_{gSTP} \frac{P_d}{P_{STP}} = 0.0753 \frac{P_d}{14.7} = \underline{\hspace{2cm}}$$

d-e Vertical Pipe

$$\Delta z = 50 \text{ ft}$$

$$\Delta P_{vert} = (4f + \lambda_z \mu) \frac{L}{D} \frac{\rho_g V_g^2}{2g_c} + \rho^0 \Delta z \frac{g}{g_c}$$

$$f = \underline{\hspace{2cm}}$$

$$Fr = \underline{\hspace{2cm}}$$

$$\lambda_z = \underline{\hspace{2cm}}$$

$$\rho^0 = \varepsilon \rho_g + (1 - \varepsilon) \rho_p = \underline{\hspace{2cm}}$$

$$\varepsilon = 1 - \frac{w_s}{A \rho_p V_p} = \underline{\hspace{2cm}}$$

$$\frac{V_p}{V_g} = 1 - 0.123 d_p^{0.3} \rho_p^{0.5} \quad V_p = \underline{\hspace{2cm}}$$

$\mu = 4.833$ from previous page
(loading is constant)

$B = 0.5$ for this R/D

$$\begin{aligned} \Delta P_{cd} &= 0.5(1 + 4.833) \\ &\times \frac{0.0850(103.3)^2}{2 \cdot 32.174} \frac{1 \text{ ft}^2}{144 \text{ in}^2} \\ &= 0.286 \text{ psi} \end{aligned}$$

$$P_d = 16.61 + 0.286 = 16.89 \text{ psia}$$

$$V_g = 101.55 \text{ ft/s}$$

$$\rho_g = 0.0850 \text{ lbm/ft}^3$$

Note $\rho_g V_g = \text{constant}$

Hence, Re does not change.

Since $f = f(\text{Re}, k/D)$ then f does not change.

$$f = 0.00419$$

$$\mu = \text{constant} = 4.833$$

$$Fr_s = \text{constant} = 2218$$

$$Fr = \frac{(101.55 \text{ ft/s})^2}{32.174 \text{ ft/s}^2 \cdot 0.5 \text{ ft}} = 641.05$$

$$\begin{aligned} \lambda_z &= 0.082 \mu^{-0.3} Fr^{-0.86} Fr_s^{0.25} \left(\frac{D}{d_p} \right)^{0.1} \\ &= 0.001946 \end{aligned}$$

$$\begin{aligned} \frac{V_p}{V_g} &= 1 - 0.123(0.01312)^{0.3} 59^{0.5} \\ &= 0.7425 \end{aligned}$$

$$V_p = 75.40 \text{ ft/s}$$

$$\begin{aligned} \varepsilon &= 1 - \frac{30,000 \text{ lbm/hr} (1 \text{ hr} / 3600 \text{ s})}{0.1963 \text{ ft}^2 \cdot 59 \text{ lbm/ft}^3 \cdot 75.40 \text{ ft/s}} \\ &= 0.9905 \end{aligned}$$

$$\begin{aligned} \rho^0 &= 0.9905(0.0850) + (1 - 0.9905)59 \\ &= 0.6485 \text{ lbm/ft}^3 \end{aligned}$$

$$\begin{aligned} \Delta P_{de} &= (4 \cdot 0.00419 + 0.001946 \cdot 4.833) \frac{50}{0.5} \frac{0.0850(101.55)^2}{2 \cdot 32.174} \\ &+ 0.6485 \frac{32.174}{32.174} 50 \\ &= 35.64 + 32.43 \text{ lbf / ft}^2 \\ &= 0.465 \text{ psi} \end{aligned}$$

At point 'e'

$$P_e = \underline{\hspace{2cm}}$$

$$P_e = 0.465 + 16.89 = 17.36 \text{ psia}$$

$$V_g = \frac{Q_{STP}}{A} \frac{P_{STP}}{P_e} = \frac{1375 \text{ ft}^3 / \text{min}}{0.1963 \text{ ft}^2} \frac{14.7 \text{ psi}}{P_e} \frac{\text{min}}{60s} = \underline{\hspace{2cm}}$$

$$V_g = 98.83 \text{ ft/s}$$

$$\rho_g = \rho_{gSTP} \frac{P_e}{P_{STP}} = 0.0753 \frac{P_e}{14.7} = \underline{\hspace{2cm}}$$

$$\rho_g = 0.0889 \text{ lbm/ft}^3$$

e-f Pipe Bend

$$\Delta P_{bend} = B(1 + \mu) \frac{\rho_g V_g^2}{2g_c}$$

$$\Delta P_{cd} = \underline{\hspace{2cm}}$$

$$\Delta P_{ef} = 0.5(1 + 4.833)$$

$$\times \frac{0.0889(98.83)^2}{2 \cdot 32.174} \frac{1 \text{ ft}^2}{144 \text{ in}^2}$$

$$= 0.273 \text{ psi}$$

At point 'f'

$$P_f = \underline{\hspace{2cm}}$$

$$P_f = 17.36 + 0.273 = 17.63 \text{ psia}$$

$$V_g = \frac{Q_{STP}}{A} \frac{P_{STP}}{P_d} = \frac{1375 \text{ ft}^3 / \text{min}}{0.1963 \text{ ft}^2} \frac{14.7 \text{ psi}}{P_d} \frac{\text{min}}{60s} = \underline{\hspace{2cm}}$$

$$V_g = 97.30 \text{ ft/s}$$

$$\rho_g = \rho_{gSTP} \frac{P_d}{P_{STP}} = 0.0753 \frac{P_d}{14.7} = \underline{\hspace{2cm}}$$

$$\rho_g = 0.0903 \text{ lbm/ft}^3$$

f-g Horizontal Pipe

L = 100 ft plus has acceleration zone

These calculations are similar to pipe section b-c plus the addition of the acceleration

$$\Delta P_{fg} = \Delta P_{horiz} + \Delta P_{accel}$$

$$\Delta P_{horiz} = (4f + \lambda_z \mu) \frac{L}{D} \frac{\rho_g V_g^2}{2g_c} = \underline{\hspace{2cm}}$$

Intermediate value for P_g (used for calculating V_g that is used in calculating the ΔP_{accel})

$$P_g' = P_f + \Delta P_{horiz} =$$

$$V_g = \frac{Q_{STP}}{A} \frac{P_{STP}}{P_d} = \frac{1375 \text{ ft}^3 / \text{min}}{0.1963 \text{ ft}^2} \frac{14.7 \text{ psi}}{P_d} \frac{\text{min}}{60s} = \underline{\hspace{2cm}}$$

$$\rho_g = \rho_{gSTP} \frac{P_d}{P_{STP}} = 0.0753 \frac{P_d}{14.7} = \underline{\hspace{2cm}}$$

$$\Delta P_{accel} = \frac{\rho_g V_g^2}{2g_c} \left(1 + 2\mu \left(\frac{V_p}{V_g} \right) \right) = \underline{\hspace{2cm}}$$

$$P_g = \underline{\hspace{2cm}}$$

Pressure Drop Across Blower

Inlet pressure to blower

$$P_{in} = 14.7 - 0.3 = 14.4 \text{ psia}$$

$$\Delta P_{blower} = P_g - P_{in} = \underline{\hspace{2cm}}$$

This is the pressure increase required by the blower.

$$Fr = \frac{(97.30 \text{ ft/s})^2}{32.174 \text{ ft/s}^2 \cdot 0.5 \text{ ft}} = 588.49$$

$$\lambda_z = 0.082 \mu^{-0.3} Fr^{-0.86} Fr_s^{0.25} \left(\frac{D}{d_p} \right)^{0.1} = 0.002095$$

$$\Delta P_{horiz} = (4 \cdot 0.00419 + 0.002095 \cdot 4.833) \times \frac{100}{0.5} \frac{0.0903 \text{ lbm/ft}^3 (97.3 \text{ ft/s})^2}{2 \cdot 32.774 \frac{\text{lbm ft}}{\text{bf s}^2}} \times \frac{\text{ft}^2}{144 \text{ in}^2} = 0.473 \text{ psi}$$

$$P_g' = 17.63 + 0.473 = 18.11 \text{ psia}$$

$$V_g = 94.76 \text{ ft/s}$$

$$\rho_g = 0.0927 \text{ lbm/ft}^3$$

$$V_p/V_g = 0.7425 \text{ (from previous)}$$

$$\Delta P_{accel} = 0.734 \text{ psi}$$

$$P_g = 18.84 \text{ psia}$$

$$\Delta P_{blower} = 18.84 - 14.4 = 4.44 \text{ psi}$$

Check Saltation Velocity

Use the Rizk Correlation

$$\delta = 1.44d_p + 1.96$$

$$\chi = 1.1d_p + 2.5$$

$$V_{gs} = \left[\frac{w_s 10^\delta}{A \rho_g} (\sqrt{gD})^\chi \right]^{\left(\frac{1}{\chi+1}\right)}$$

Note, d_p is in mm

From the Rizk equation, the largest saltation velocity occurs at the smallest gas density (at the exit of the cyclone). Use this condition to calculate the saltation velocity.

$$\delta = \underline{\hspace{2cm}}$$

$$\chi = \underline{\hspace{2cm}}$$

$$\delta = 1.44(4mm) + 1.96 = 7.72$$

$$\chi = 1.1(4mm) + 2.5 = 6.90$$

$$V_{gs} = \left[\frac{30000 \frac{lbm}{hr} \frac{hr}{3600s} 10^{7.72}}{0.1964 ft^2 \cdot 0.075 \frac{lbm}{ft^3}} \left(\sqrt{32.174 \frac{ft}{s^2} \cdot 0.5 ft} \right)^{6.9} \right]^{\left(\frac{1}{6.9+1}\right)}$$
$$= 71.2 \text{ ft/s}$$

The smallest velocity in the pipeline occurs at point g (94.76 ft/s) hence the velocity everywhere in the pipeline exceeds the saltation velocity.

Assuming the blower is capable of the 4.44 psid pressure increase, the velocity provided by the flow rate of 1375 SCFM exceeds the saltation velocity everywhere in the pipe line, hence the blower and pipe system is capable of conveying 30,000lbm/hr of solids.