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Time-dependent Green's functions for an anisotropic bimaterial with viscous interface

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Abstract

By virtue of the Stroh formalism, we derive the exact closed-form solutions for the time-dependent two-dimensional Green's functions due to a line force and line dislocation in an anisotropic bimaterial with a viscous interface. We first reduce the boundary value problem to two coupled homogeneous first-order partial differential equations, which can be solved using a decoupling technique. The full-field expressions of the time-dependent displacements and stresses due to the line force and line dislocation interacting with the viscous interface are obtained.

Keywords: Anisotropic bimaterial; Green's function; Viscous interface; Stroh formalism

1. Introduction

The influence of the interfacial viscosity in composites should be taken into consideration under some situations. For example viscous interfacial layers are often introduced artificially to tailor the mechanical properties of laminates, such as the damping performances. On the viscous interfacial layer, the sliding rate $\dot{\delta}$ and the interfacial shear stress τ obey the linear law

$$\dot{\delta} = \frac{\tau}{\eta}, \tag{1}$$

where a dot over the quantity denotes differentiation with respect to time t, and η is the interfacial viscosity which can be determined experimentally. Ray and Ashby (1971) and Suo (1997) suggested that the microscopically diffusion-controlled sliding mechanism can also be macroscopically described by Eq. (1).

Based on the interface model described by Eq. (1), He (2001) and He and Lim (2001) studied the time-dependent mechanical responses of the particle- and fiber-reinforced composites with the viscous interface. It is found that significant stress relaxation occurs, and the effective elastic moduli of the composites change remarkably with time. He and Liu (2005) analyzed the mechanical damping of fiber composites with viscous interface. He and Jiang (2003) studied the time-dependent mechanical responses of laminated strips with linearly viscous interfaces in cylindrical

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bending. Chen and Lee (2004) investigated a simply supported angle-ply laminated plate in cylindrical bending with viscous interfaces under static loads based on state-space formulations. As far as we know, the Green's function solutions for an anisotropic bimaterial with viscous interface have not been recorded in the open literature. The results of Barnett et al. (1998) show that in an elastically *anisotropic* medium second slip-wave modes can exist for many orientations of slip surface and wave direction in the slip surface. Such slip modes are not possible in an isotropic medium; they are purely *anisotropy* effects.

In this work, we study the time-dependent mechanical response due to a line force and line dislocation in an anisotropic bimaterial with a viscous interface. The analysis is restricted to a quasi-static process in which the inertia force in the two anisotropic half-planes is ignored. As a result the Stroh formalism for two-dimensional (2D) anisotropic elasticity (Stroh, 1958; Ting, 1996) can be conveniently employed.

2. The Stroh formalism

In a fixed Cartesian coordinate system x_i $(i=1\text{–}3)$, we let u_i and σ_{ij} be the elastic displacement and stress in an anisotropic elastic material. The equations of equilibrium and the stress–strain law are

$$\sigma_{ij,j}=0, \qquad \sigma_{ij}=C_{ijks}u_{k,s}, \tag{2}$$

where the subscript comma denotes differentiation, and C_{ijks} are the elastic constants. Here the inertial force of the anisotropic material is ignored. For a 2D deformation in which u_i depends on x_1 and x_2 only, one can seek the solution in the form (Stroh, 1958; Ting, 1996)

$$\mathbf{u}=[u_1 \quad u_2 \quad u_3]^{\mathrm{T}}=\mathbf{a}f(x_1+px_2,t), \tag{3}$$

where the superscript T denotes transpose, $\mathbf{a}$ is a 3×1 column, p is a complex number, $f(\cdot,t)$ an analytic function. The appearance of the time t is due to the influence of the viscous interface. Thus all equations in (2) are satisfied for the arbitrary analytic function $f(\cdot,t)$ if (Ting, 1996)

$$\left[\mathbf{Q}+p\left(\mathbf{R}+\mathbf{R}^{\mathrm{T}}\right)+p^2\mathbf{T}\right]\mathbf{a}=\mathbf{0}, \tag{4}$$

where the 3×3 real matrix $\mathbf{R}$ and the two 3×3 symmetric matrices $\mathbf{Q}$ and $\mathbf{T}$ are defined by

$$Q_{ik}=C_{i1k1}, \quad R_{ik}=C_{i1k2}, \quad T_{ik}=C_{i2k2}. \tag{5}$$

For a stable material with positive-definite energy density, the six roots of (4) form three distinct conjugate pairs with non-zero imaginary parts. Let p_i $(i=1\text{–}3)$ be the three distinct roots with positive imaginary parts and $\mathbf{a}_i$ the associated eigenvectors, then the general solution is given by (Ting, 1996)

$$\begin{aligned}&\mathbf{u}=[u_1 \quad u_2 \quad u_3]^{\mathrm{T}}=\mathbf{Af}(z,t)+\bar{\mathbf{A}}\overline{\mathbf{f}(z,t)},\\ &\boldsymbol{\Phi}=[\phi_1 \quad \phi_2 \quad \phi_3]^{\mathrm{T}}=\mathbf{Bf}(z,t)+\bar{\mathbf{B}}\overline{\mathbf{f}(z,t)},\end{aligned} \tag{6}$$

where ϕ_i $(i=1\text{–}3)$ are the three stress functions, and

$$\begin{aligned}&\mathbf{b}_i=\left(\mathbf{R}^{\mathrm{T}}+p_i\mathbf{T}\right)\mathbf{a}_i=\frac{-1}{p_i}(\mathbf{Q}+p_i\mathbf{R})\mathbf{a}_i \quad (i=1\text{–}3),\\ &\mathbf{A}=[\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_3], \qquad \mathbf{B}=[\mathbf{b}_1 \quad \mathbf{b}_2 \quad \mathbf{b}_3],\\ &\mathbf{f}(z,t)=[f_1(z_1,t) \quad f_2(z_2,t) \quad f_3(z_3,t)]^{\mathrm{T}},\\ &z_i=x_1+p_ix_2, \quad \mathrm{Im}\{p_i\}>0 \quad (i=1\text{–}3).\end{aligned} \tag{7}$$

Thus the stresses are given by

$$\sigma_{i1}=-\phi_{i,2}, \quad \sigma_{i2}=\phi_{i,1} \quad (i=1\text{–}3). \tag{8}$$

The two matrices $\mathbf{A}$ and $\mathbf{B}$ satisfy the following normalized orthogonal relationship (Ting, 1996)

$$\begin{bmatrix}\mathbf{B}^{\mathrm{T}} & \mathbf{A}^{\mathrm{T}}\\ \bar{\mathbf{B}}^{\mathrm{T}} & \bar{\mathbf{A}}^{\mathrm{T}}\end{bmatrix}\begin{bmatrix}\mathbf{A} & \bar{\mathbf{A}}\\ \mathbf{B} & \bar{\mathbf{B}}\end{bmatrix}=\mathbf{I}. \tag{9}$$

Furthermore, the following three real matrices $\mathbf{S}$, $\mathbf{H}$ and $\mathbf{L}$ can be introduced (Ting, 1996)

$$\mathbf{S}=\mathrm{i}\left(2\mathbf{AB}^{\mathrm{T}}-\mathbf{I}\right), \quad \mathbf{H}=2\mathrm{i}\mathbf{AA}^{\mathrm{T}}, \quad \mathbf{L}=-2\mathrm{i}\mathbf{BB}^{\mathrm{T}} \tag{10}$$

with $\mathbf{H}$ and $\mathbf{L}$ being symmetric, and $\mathbf{SH}, \mathbf{LS}, \mathbf{H}^{-1}\mathbf{S}, \mathbf{SL}^{-1}$ being anti-symmetric.

3. Green's function solutions

Two jointed semi-infinite anisotropic elastic solids consist of solid 1 ($x_2 > 0$) with elastic constants $C_{ijkl}^{(1)}$ and solid 2 ($x_2 < 0$) with elastic constants $C_{ijkl}^{(2)}$. Assume that a line force $\hat{\mathbf{f}}$ and a line dislocation with Burgers vector $\hat{\mathbf{b}}$ are located at $[\hat{x}_1, \hat{x}_2]$ ($\hat{x}_2 > 0$) in the upper half-plane of the bimaterial. Throughout this paper, the subscripts 1 and 2 (or the superscripts (1) and (2)) are used to identify the quantities in the upper and lower half-planes, respectively. The two anisotropic half-planes are bonded together through a viscous interface $x_2 = 0$. The boundary conditions on the viscous interface can be expressed as (see He and Jiang, 2003, Chen and Lee, 2004)

$$\begin{aligned}&\sigma_{12}^{(1)} = \sigma_{12}^{(2)}, \quad \sigma_{22}^{(1)} = \sigma_{22}^{(2)}, \quad \sigma_{32}^{(1)} = \sigma_{32}^{(2)}, \\ &u_2^{(1)} = u_2^{(2)}, \qquad\qquad\qquad\qquad x_2 = 0 \text{ and } t > 0, \\ &\sigma_{12}^{(2)} = \eta_1\big(\dot{u}_1^{(1)} - \dot{u}_1^{(2)}\big), \quad \sigma_{32}^{(2)} = \eta_3\big(\dot{u}_3^{(1)} - \dot{u}_3^{(2)}\big),\end{aligned} \tag{11}$$

where η_1 and η_3 are the viscous coefficients in the x_1 and x_3 directions, respectively. At $t = 0$ when the line force and line dislocation are just introduced into the upper half-plane, the displacements across the interface have no time to experience any jump from the dashpot. Therefore at the initial time $t = 0$ the interface is a perfect one. In addition we ignore the inertia force in the two anisotropic half-planes, and, as a result the Stroh formalism outlined in the previous section can still be conveniently adopted to address this problem. In the following derivations, we will replace the complex variables z_j ($j = 1$–3) by the common complex variable $z = x_1 + \mathrm{i}x_2$ due to the fact that $z_1 = z_2 = z_3 = z$ on the real axis (Clements, 1971). When the analysis is finished, the complex variable $z = x_1 + \mathrm{i}x_2$ shall be changed back to the corresponding complex variables z_j ($j = 1$–3). To facilitate the analysis, we introduce two new analytic function vectors, $\mathbf{h}(z,t)$ defined in the upper half-plane and $\mathbf{g}(z,t)$ defined in the lower half-plane as

$$\begin{aligned}&\mathbf{h}(z,t) = [h_1(z,t) \quad h_2(z,t) \quad h_3(z,t)]^{\mathrm{T}} = \mathbf{B}_1\mathbf{f}_1(z,t), \\ &\mathbf{g}(z,t) = [g_1(z,t) \quad g_2(z,t) \quad g_3(z,t)]^{\mathrm{T}} = \mathbf{B}_2\mathbf{f}_2(z,t).\end{aligned} \tag{12}$$

Eq. $(11)_1$ can be expressed in terms of the two analytic function vectors $\mathbf{h}(z,t)$ and $\mathbf{g}(z,t)$ as

$$\mathbf{h}^+(x_1,t) + \bar{\mathbf{h}}^-(x_1,t) = \mathbf{g}^-(x_1,t) + \bar{\mathbf{g}}^+(x_1,t), \quad x_2 = 0. \tag{13}$$

It follows from the above expression that

$$\begin{aligned}&\mathbf{h}(z,t) = \bar{\mathbf{g}}(z,t) + \mathbf{h}_0(z) - \bar{\mathbf{h}}_0(z), \\ &\bar{\mathbf{h}}(z,t) = \mathbf{g}(z,t) - \mathbf{h}_0(z) + \bar{\mathbf{h}}_0(z),\end{aligned} \tag{14}$$

where

$$\mathbf{h}_0(z) = [h_{10}(z) \quad h_{20}(z) \quad h_{30}(z)]^{\mathrm{T}} = \frac{1}{2\pi\mathrm{i}}\mathbf{B}_1 < \ln(z - \hat{z}_\alpha) > \big(\mathbf{A}_1^{\mathrm{T}}\hat{\mathbf{f}} + \mathbf{B}_1^{\mathrm{T}}\hat{\mathbf{b}}\big), \tag{15}$$

and

$$\big\langle \ln(z - \hat{z}_\alpha)\big\rangle = \operatorname{diag}\big[\ln(z - \hat{z}_1) \quad \ln(z - \hat{z}_2) \quad \ln(z - \hat{z}_3)\big], \tag{16}$$

with $\hat{z}_\alpha = \hat{x}_1 + p_\alpha \hat{x}_2$.

Eq. $(11)_2$ can be expressed in terms of the two analytic function vectors $\mathbf{h}(z,t)$ and $\mathbf{g}(z,t)$ as

$$\mathbf{J}_2\mathbf{M}_1\mathbf{h}^+(x_1,t) - \mathbf{J}_2\bar{\mathbf{M}}_1\bar{\mathbf{h}}^-(x_1,t) = \mathbf{J}_2\mathbf{M}_2\mathbf{g}^-(x_1,t) - \mathbf{J}_2\bar{\mathbf{M}}_2\bar{\mathbf{g}}^+(x_1,t), \quad x_2 = 0, \tag{17}$$

where $\mathbf{J}_2 = [0 \quad 1 \quad 0]$, and $\mathbf{M}_1$ and $\mathbf{M}_2$ are two 3×3 positive definite Hermitian matrices given by

$$\mathbf{M}_k = \bar{\mathbf{M}}_k^{\mathrm{T}} = \begin{bmatrix} M_{11}^{(k)} & M_{12}^{(k)} & M_{13}^{(k)} \\ \bar{M}_{12}^{(k)} & M_{22}^{(k)} & M_{23}^{(k)} \\ \bar{M}_{13}^{(k)} & \bar{M}_{23}^{(k)} & M_{33}^{(k)} \end{bmatrix} = \mathrm{i}\mathbf{A}_k\mathbf{B}_k^{-1} = (\mathbf{I} - \mathrm{i}\mathbf{S}_k)\mathbf{L}_k^{-1} \quad (k = 1,2). \tag{18}$$

It follows from the above expression that

$$\begin{aligned}&\mathbf{J}_2\mathbf{M}_1\mathbf{h}(z,t) = -\mathbf{J}_2\bar{\mathbf{M}}_2\bar{\mathbf{g}}(z,t) + \mathbf{J}_2\mathbf{M}_1\mathbf{h}_0(z) + \mathbf{J}_2\bar{\mathbf{M}}_1\bar{\mathbf{h}}_0(z), \\ &\mathbf{J}_2\bar{\mathbf{M}}_1\bar{\mathbf{h}}(z,t) = -\mathbf{J}_2\mathbf{M}_2\mathbf{g}(z,t) + \mathbf{J}_2\mathbf{M}_1\mathbf{h}_0(z) + \mathbf{J}_2\bar{\mathbf{M}}_1\bar{\mathbf{h}}_0(z).\end{aligned} \tag{19}$$

In view of Eqs. (14) and (19), the four analytic functions $h_j(z,t)$ $(j=1\text{–}3)$ and $\bar{g}_2(z,t)$ defined in the upper half-plane can be expressed in terms of the two analytic functions $\bar{g}_1(z,t)$ and $\bar{g}_3(z,t)$ also defined in the upper half-plane as

$$\begin{aligned}
h_1(z,t) &= \bar{g}_1(z,t) + h_{10}(z) - \bar{h}_{10}(z),\\
h_2(z,t) &= -\frac{\bar{M}_{12}^{(1)} + M_{12}^{(2)}}{M_{22}^{(1)} + M_{22}^{(2)}}\bar{g}_1(z,t) - \frac{M_{23}^{(1)} + \bar{M}_{23}^{(2)}}{M_{22}^{(1)} + M_{22}^{(2)}}\bar{g}_3(z,t) + h_{20}(z)\\
&\quad + \frac{M_{12}^{(1)} + \bar{M}_{12}^{(1)}}{M_{22}^{(1)} + M_{22}^{(2)}}\bar{h}_{10}(z) + \frac{M_{22}^{(1)} - M_{22}^{(2)}}{M_{22}^{(1)} + M_{22}^{(2)}}\bar{h}_{20}(z) + \frac{M_{23}^{(1)} + \bar{M}_{23}^{(1)}}{M_{22}^{(1)} + M_{22}^{(2)}}\bar{h}_{30}(z),\\
h_3(z,t) &= \bar{g}_3(z,t) + h_{30}(z) - \bar{h}_{30}(z),\\
\bar{g}_2(z,t) &= -\frac{\bar{M}_{12}^{(1)} + M_{12}^{(2)}}{M_{22}^{(1)} + M_{22}^{(2)}}\bar{g}_1(z,t) - \frac{M_{23}^{(1)} + \bar{M}_{23}^{(2)}}{M_{22}^{(1)} + M_{22}^{(2)}}\bar{g}_3(z,t)\\
&\quad + \frac{M_{12}^{(1)} + \bar{M}_{12}^{(1)}}{M_{22}^{(1)} + M_{22}^{(2)}}\bar{h}_{10}(z) + \frac{2M_{22}^{(1)}}{M_{22}^{(1)} + M_{22}^{(2)}}\bar{h}_{20}(z) + \frac{M_{23}^{(1)} + \bar{M}_{23}^{(1)}}{M_{22}^{(1)} + M_{22}^{(2)}}\bar{h}_{30}(z).
\end{aligned}\tag{20}$$

Similarly the four analytic functions $\bar{h}_j(z,t)$ $(j=1\text{–}3)$ and $g_2(z,t)$ defined in the lower half-plane can be expressed in terms of the two analytic functions $g_1(z,t)$ and $g_3(z,t)$ also defined in the lower half-plane as

$$\begin{aligned}
\bar{h}_1(z,t) &= g_1(z,t) + \bar{h}_{10}(z) - h_{10}(z),\\
\bar{h}_2(z,t) &= -\frac{M_{12}^{(1)} + \bar{M}_{12}^{(2)}}{M_{22}^{(1)} + M_{22}^{(2)}}g_1(z,t) - \frac{\bar{M}_{23}^{(1)} + M_{23}^{(2)}}{M_{22}^{(1)} + M_{22}^{(2)}}g_3(z,t) + \bar{h}_{20}(z)\\
&\quad + \frac{M_{12}^{(1)} + \bar{M}_{12}^{(1)}}{M_{22}^{(1)} + M_{22}^{(2)}}h_{10}(z) + \frac{M_{22}^{(1)} - M_{22}^{(2)}}{M_{22}^{(1)} + M_{22}^{(2)}}h_{20}(z) + \frac{M_{23}^{(1)} + \bar{M}_{23}^{(1)}}{M_{22}^{(1)} + M_{22}^{(2)}}h_{30}(z),\\
\bar{h}_3(z,t) &= g_3(z,t) + \bar{h}_{30}(z) - h_{30}(z),\\
g_2(z,t) &= -\frac{M_{12}^{(1)} + \bar{M}_{12}^{(2)}}{M_{22}^{(1)} + M_{22}^{(2)}}g_1(z,t) - \frac{\bar{M}_{23}^{(1)} + M_{23}^{(2)}}{M_{22}^{(1)} + M_{22}^{(2)}}g_3(z,t)\\
&\quad + \frac{M_{12}^{(1)} + \bar{M}_{12}^{(1)}}{M_{22}^{(1)} + M_{22}^{(2)}}h_{10}(z) + \frac{2M_{22}^{(1)}}{M_{22}^{(1)} + M_{22}^{(2)}}h_{20}(z) + \frac{M_{23}^{(1)} + \bar{M}_{23}^{(1)}}{M_{22}^{(1)} + M_{22}^{(2)}}h_{30}(z).
\end{aligned}\tag{21}$$

Eq. $(11)_3$ can be expressed in terms of $\mathbf{h}(z,t)$ and $\mathbf{g}(z,t)$ as follows

$$\begin{aligned}
&-\mathrm{i}\begin{bmatrix} M_{11}^{(1)} & M_{12}^{(1)} & M_{13}^{(1)}\\ \bar{M}_{13}^{(1)} & \bar{M}_{23}^{(1)} & M_{33}^{(1)}\end{bmatrix}\frac{\partial \mathbf{h}^{+}(x_1,t)}{\partial t} + \mathrm{i}\begin{bmatrix} M_{11}^{(1)} & \bar{M}_{12}^{(1)} & \bar{M}_{13}^{(1)}\\ M_{13}^{(1)} & M_{23}^{(1)} & M_{33}^{(1)}\end{bmatrix}\frac{\partial \bar{\mathbf{h}}^{-}(x_1,t)}{\partial t}\\
&\quad +\mathrm{i}\begin{bmatrix} M_{11}^{(2)} & M_{12}^{(2)} & M_{13}^{(2)}\\ \bar{M}_{13}^{(2)} & \bar{M}_{23}^{(2)} & M_{33}^{(2)}\end{bmatrix}\frac{\partial \mathbf{g}^{-}(x_1,t)}{\partial t} - \mathrm{i}\begin{bmatrix} M_{11}^{(2)} & \bar{M}_{12}^{(2)} & \bar{M}_{13}^{(2)}\\ M_{13}^{(2)} & M_{23}^{(2)} & M_{33}^{(2)}\end{bmatrix}\frac{\partial \bar{\mathbf{g}}^{+}(x_1,t)}{\partial t} \qquad x_2 = 0\\
&= \begin{bmatrix} \eta_1^{-1} & 0\\ 0 & \eta_3^{-1}\end{bmatrix}\begin{bmatrix} \frac{\partial g_1^{-}(x_1,t)}{\partial x_1} + \frac{\partial \bar{g}_1^{+}(x_1,t)}{\partial x_1}\\ \frac{\partial g_3^{-}(x_1,t)}{\partial x_1} + \frac{\partial \bar{g}_3^{+}(x_1,t)}{\partial x_1}\end{bmatrix}.
\end{aligned}\tag{22}$$

Substituting the results of Eqs. (20) and (21) into Eq. (22), we finally obtain

$$\begin{aligned}
&\begin{bmatrix} \eta_1^{-1} & 0\\ 0 & \eta_3^{-1}\end{bmatrix}\begin{bmatrix} \frac{\partial g_1^{-}(x_1,t)}{\partial x_1}\\ \frac{\partial g_3^{-}(x_1,t)}{\partial x_1}\end{bmatrix} - \mathrm{i}\begin{bmatrix} H_{11} & H_{12}\\ \bar{H}_{12} & H_{22}\end{bmatrix}\begin{bmatrix} \frac{\partial g_1^{-}(x_1,t)}{\partial t}\\ \frac{\partial g_3^{-}(x_1,t)}{\partial t}\end{bmatrix}\\
&= -\begin{bmatrix} \eta_1^{-1} & 0\\ 0 & \eta_3^{-1}\end{bmatrix}\begin{bmatrix} \frac{\partial \bar{g}_1^{+}(x_1,t)}{\partial x_1}\\ \frac{\partial \bar{g}_3^{+}(x_1,t)}{\partial x_1}\end{bmatrix} - \mathrm{i}\begin{bmatrix} H_{11} & \bar{H}_{12}\\ H_{12} & H_{22}\end{bmatrix}\begin{bmatrix} \frac{\partial \bar{g}_1^{+}(x_1,t)}{\partial t}\\ \frac{\partial \bar{g}_3^{+}(x_1,t)}{\partial t}\end{bmatrix},
\end{aligned} \qquad x_2 = 0 \tag{23}$$

where

$$
\begin{aligned}
H_{11} &= M_{11}^{(1)} + M_{11}^{(2)} - \frac{(M_{12}^{(1)} + \bar{M}_{12}^{(2)})(\bar{M}_{12}^{(1)} + M_{12}^{(2)})}{M_{22}^{(1)} + M_{22}^{(2)}},\\
H_{22} &= M_{33}^{(1)} + M_{33}^{(2)} - \frac{(M_{23}^{(1)} + \bar{M}_{23}^{(2)})(\bar{M}_{23}^{(1)} + M_{23}^{(2)})}{M_{22}^{(1)} + M_{22}^{(2)}},\\
H_{12} &= M_{13}^{(1)} + \bar{M}_{13}^{(2)} - \frac{(\bar{M}_{12}^{(1)} + M_{12}^{(2)})(\bar{M}_{23}^{(1)} + M_{23}^{(2)})}{M_{22}^{(1)} + M_{22}^{(2)}}.
\end{aligned}
\tag{24}
$$

It can be easily shown that $H_{11} > 0$, $H_{22} > 0$, $H_{11}H_{22} - H_{12}\bar{H}_{12} > 0$. It is observed that the left-hand side of Eq. (23) is analytic in the lower half-plane including the point at infinity, while the right-hand side of Eq. (23) is analytic in the upper half-plane including the point at infinity. By employing the Liouville's theorem, the left- and right-hand sides should be identically zero. It follows that

$$
\begin{bmatrix} \eta_1^{-1} & 0 \\ 0 & \eta_3^{-1} \end{bmatrix}
\begin{bmatrix} \frac{\partial g_1(z,t)}{\partial z} \\ \frac{\partial g_3(z,t)}{\partial z} \end{bmatrix}
- \mathrm{i} \begin{bmatrix} H_{11} & H_{12} \\ \bar{H}_{12} & H_{22} \end{bmatrix}
\begin{bmatrix} \frac{\partial g_1(z,t)}{\partial t} \\ \frac{\partial g_3(z,t)}{\partial t} \end{bmatrix}
= \begin{bmatrix} 0 \\ 0 \end{bmatrix}.
\tag{25}
$$

Eq. (25) is a set of two coupled homogeneous, first-order partial differential equations for the two analytic functions $g_1(z,t)$ and $g_3(z,t)$. In order to solve the above equations, we consider the following eigenvalue problem

$$
\begin{bmatrix} H_{11} & H_{12} \\ \bar{H}_{12} & H_{22} \end{bmatrix} \mathbf{v} = \lambda \begin{bmatrix} \eta_1^{-1} & 0 \\ 0 & \eta_3^{-1} \end{bmatrix} \mathbf{v}.
\tag{26}
$$

The two real eigenvalues of this eigenvalue problem can be determined to be

$$
\begin{aligned}
\lambda_1 &= \frac{H_{11}\eta_1 + H_{22}\eta_3 + \sqrt{(H_{11}\eta_1 - H_{22}\eta_3)^2 + 4H_{12}\bar{H}_{12}\eta_1\eta_3}}{2} > 0,\\
\lambda_2 &= \frac{H_{11}\eta_1 + H_{22}\eta_3 - \sqrt{(H_{11}\eta_1 - H_{22}\eta_3)^2 + 4H_{12}\bar{H}_{12}\eta_1\eta_3}}{2} > 0.
\end{aligned}
\tag{27}
$$

The corresponding two eigenvectors $\mathbf{v}_1$ and $\mathbf{v}_2$ associated with the two eigenvalues λ_1 and λ_2 can be determined as

$$
\mathbf{v}_1 = \begin{bmatrix} H_{12}\eta_1 \\ \lambda_1 - H_{11}\eta_1 \end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix} H_{12}\eta_1 \\ \lambda_2 - H_{11}\eta_1 \end{bmatrix}.
\tag{28}
$$

Apparently the following orthogonal relationships with respect to the two matrices

$$
\begin{bmatrix} \eta_1^{-1} & 0 \\ 0 & \eta_3^{-1} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} H_{11} & H_{12} \\ \bar{H}_{12} & H_{22} \end{bmatrix}
$$

hold

$$
\bar{\boldsymbol{\Omega}}^{\mathrm{T}} \begin{bmatrix} \eta_1^{-1} & 0 \\ 0 & \eta_3^{-1} \end{bmatrix} \boldsymbol{\Omega} = \boldsymbol{\Lambda}_2, \quad \bar{\boldsymbol{\Omega}}^{\mathrm{T}} \begin{bmatrix} H_{11} & H_{12} \\ \bar{H}_{12} & H_{22} \end{bmatrix} \boldsymbol{\Omega} = \boldsymbol{\Lambda}_1\boldsymbol{\Lambda}_2,
\tag{29}
$$

where $\boldsymbol{\Omega} = [\mathbf{v}_1 \quad \mathbf{v}_2]$, and $\boldsymbol{\Lambda}_1$ and $\boldsymbol{\Lambda}_2$ are two 2×2 diagonal matrices given by

$$
\boldsymbol{\Lambda}_1 = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix},
\tag{30}
$$

$$
\boldsymbol{\Lambda}_2 = \begin{bmatrix} H_{12}\bar{H}_{12}\eta_1 + \eta_3^{-1}(\lambda_1 - H_{11}\eta_1)^2 & 0 \\ 0 & H_{12}\bar{H}_{12}\eta_1 + \eta_3^{-1}(\lambda_2 - H_{11}\eta_1)^2 \end{bmatrix}.
\tag{31}
$$

We now introduce two new analytic functions $Y_1(z,t)$ and $Y_3(z,t)$ which are related to $g_1(z,t)$ and $g_3(z,t)$ through

$$
\begin{bmatrix} g_1(z,t) \\ g_3(z,t) \end{bmatrix} = \boldsymbol{\Omega} \begin{bmatrix} Y_1(z,t) \\ Y_3(z,t) \end{bmatrix}.
\tag{32}
$$

In view of Eqs. (29) and (32), Eq. (25) can be decoupled as

$$\begin{aligned}&\frac{\partial Y_1(z,t)}{\partial z}-\mathrm{i}\lambda_1\frac{\partial Y_1(z,t)}{\partial t}=0,\\&\frac{\partial Y_3(z,t)}{\partial z}-\mathrm{i}\lambda_2\frac{\partial Y_3(z,t)}{\partial t}=0.\end{aligned}\tag{33}$$

Due to the fact that at the initial time $t=0$, the interface $x_2=0$ is perfect, then the initial value for the two analytic functions $Y_1(z,t)$ and $Y_3(z,t)$ are

$$\begin{bmatrix}Y_1(z,0)\\Y_3(z,0)\end{bmatrix}=\frac{1}{2\pi\mathrm{i}}\boldsymbol{\Omega}^{-1}\begin{bmatrix}\mathbf{J}_1\\\mathbf{J}_3\end{bmatrix}(\bar{\mathbf{M}}_1+\mathbf{M}_2)^{-1}(\mathbf{M}_1+\bar{\mathbf{M}}_1)\mathbf{B}_1<\ln(z-\hat{z}_\alpha)>\big(\mathbf{A}_1^{\mathrm{T}}\hat{\mathbf{f}}+\mathbf{B}_1^{\mathrm{T}}\hat{\mathbf{b}}\big),\tag{34}$$

where $\mathbf{J}_1=[1\quad 0\quad 0]$ and $\mathbf{J}_3=[0\quad 0\quad 1]$. As a result the solutions to the two independent first-order partial differential equations in Eq. (33) are given by

$$\begin{bmatrix}Y_1(z,t)\\Y_3(z,t)\end{bmatrix}=\frac{1}{2\pi\mathrm{i}}\sum_{k=1}^{3}\big\langle\ln(z-\hat{z}_k-\mathrm{i}t/\lambda_\beta)\big\rangle\boldsymbol{\Omega}^{-1}\begin{bmatrix}\mathbf{J}_1\\\mathbf{J}_3\end{bmatrix}(\bar{\mathbf{M}}_1+\mathbf{M}_2)^{-1}(\mathbf{M}_1+\bar{\mathbf{M}}_1)\mathbf{B}_1\mathbf{I}_k\big(\mathbf{A}_1^{\mathrm{T}}\hat{\mathbf{f}}+\mathbf{B}_1^{\mathrm{T}}\hat{\mathbf{b}}\big),\tag{35}$$

where

$$\big\langle\ln(z-\hat{z}_k-\mathrm{i}t/\lambda_\beta)\big\rangle=\begin{bmatrix}\ln(z-\hat{z}_k-\mathrm{i}t/\lambda_1)&0\\0&\ln(z-\hat{z}_k-\mathrm{i}t/\lambda_2)\end{bmatrix},\tag{36}$$

$$\mathbf{I}_1=\mathrm{diag}[1\quad 0\quad 0],\qquad \mathbf{I}_2=\mathrm{diag}[0\quad 1\quad 0],\qquad \mathbf{I}_3=\mathrm{diag}[0\quad 0\quad 1].\tag{37}$$

It follows from Eqs. (32) and (35) that the two analytic functions $g_1(z,t)$ and $g_3(z,t)$ are given by

$$\begin{bmatrix}g_1(z,t)\\g_3(z,t)\end{bmatrix}=\frac{1}{2\pi\mathrm{i}}\boldsymbol{\Omega}\sum_{k=1}^{3}\big\langle\ln(z-\hat{z}_k-\mathrm{i}t/\lambda_\beta)\big\rangle\boldsymbol{\Omega}^{-1}\begin{bmatrix}\mathbf{J}_1\\\mathbf{J}_3\end{bmatrix}(\bar{\mathbf{M}}_1+\mathbf{M}_2)^{-1}(\mathbf{M}_1+\bar{\mathbf{M}}_1)\mathbf{B}_1\mathbf{I}_k(\mathbf{A}_1^{\mathrm{T}}\hat{\mathbf{f}}+\mathbf{B}_1^{\mathrm{T}}\hat{\mathbf{b}}),\tag{38}$$

where

$$\big\langle\ln(z-\hat{z}_k-\mathrm{i}t/\lambda_\beta)\big\rangle=\begin{bmatrix}\ln(z-\hat{z}_k-\mathrm{i}t/\lambda_1)&0\\0&\ln(z-\hat{z}_k-\mathrm{i}t/\lambda_2)\end{bmatrix}.\tag{39}$$

In view of Eqs. (20), (21) and (38), the expressions of $\mathbf{h}(z,t)$ and $\mathbf{g}(z,t)$ are

$$\begin{aligned}\mathbf{g}(z,t)={}&\frac{1}{2\pi\mathrm{i}}\begin{bmatrix}0&0&0\\\frac{M_{12}^{(1)}+\bar{M}_{12}^{(1)}}{M_{22}^{(1)}+M_{22}^{(2)}}&\frac{2M_{22}^{(1)}}{M_{22}^{(1)}+M_{22}^{(2)}}&\frac{M_{23}^{(1)}+\bar{M}_{23}^{(1)}}{M_{22}^{(1)}+M_{22}^{(2)}}\\0&0&0\end{bmatrix}\mathbf{B}_1\big\langle\ln(z-\hat{z}_\alpha)\big\rangle\big(\mathbf{A}_1^{\mathrm{T}}\hat{\mathbf{f}}+\mathbf{B}_1^{\mathrm{T}}\hat{\mathbf{b}}\big)\\&+\frac{1}{2\pi\mathrm{i}}\begin{bmatrix}1&0\\-\frac{M_{12}^{(1)}+\bar{M}_{12}^{(2)}}{M_{22}^{(1)}+M_{22}^{(2)}}&-\frac{\bar{M}_{23}^{(1)}+M_{23}^{(2)}}{M_{22}^{(1)}+M_{22}^{(2)}}\\0&1\end{bmatrix}\\&\times\boldsymbol{\Omega}\sum_{k=1}^{3}\big\langle\ln(z-\hat{z}_k-\mathrm{i}t/\lambda_\beta)\big\rangle\boldsymbol{\Omega}^{-1}\begin{bmatrix}\mathbf{J}_1\\\mathbf{J}_3\end{bmatrix}(\bar{\mathbf{M}}_1+\mathbf{M}_2)^{-1}(\mathbf{M}_1+\bar{\mathbf{M}}_1)\mathbf{B}_1\mathbf{I}_k\big(\mathbf{A}_1^{\mathrm{T}}\hat{\mathbf{f}}+\mathbf{B}_1^{\mathrm{T}}\hat{\mathbf{b}}\big),\\&\mathrm{Im}\{z\}\leqslant 0,\end{aligned}\tag{40}$$

$$\begin{aligned}\mathbf{h}(z,t)={}&\frac{1}{2\pi\mathrm{i}}\begin{bmatrix}1&0&0\\-\frac{M_{12}^{(1)}+\bar{M}_{12}^{(1)}}{M_{22}^{(1)}+M_{22}^{(2)}}&\frac{M_{22}^{(2)}-M_{22}^{(1)}}{M_{22}^{(1)}+M_{22}^{(2)}}&-\frac{M_{23}^{(1)}+\bar{M}_{23}^{(1)}}{M_{22}^{(1)}+M_{22}^{(2)}}\\0&0&1\end{bmatrix}\bar{\mathbf{B}}_1\big\langle\ln(z-\bar{\hat{z}}_\alpha)\big\rangle\big(\bar{\mathbf{A}}_1^{\mathrm{T}}\hat{\mathbf{f}}+\bar{\mathbf{B}}_1^{\mathrm{T}}\hat{\mathbf{b}}\big)\\&+\frac{1}{2\pi\mathrm{i}}\begin{bmatrix}-1&0\\\frac{\bar{M}_{12}^{(1)}+M_{12}^{(2)}}{M_{22}^{(1)}+M_{22}^{(2)}}&\frac{M_{23}^{(1)}+\bar{M}_{23}^{(2)}}{M_{22}^{(1)}+M_{22}^{(2)}}\\0&-1\end{bmatrix}\end{aligned}$$

$$\begin{aligned}
&\times \bar{\boldsymbol{\Omega}} \sum_{k=1}^{3} \langle \ln(z - \bar{\hat{z}}_k + \mathrm{i}t/\lambda_\beta) \rangle \bar{\boldsymbol{\Omega}}^{-1} \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{J}_3 \end{bmatrix} (\mathbf{M}_1 + \bar{\mathbf{M}}_2)^{-1} (\mathbf{M}_1 + \bar{\mathbf{M}}_1) \bar{\mathbf{B}}_1 \mathbf{I}_k \big(\bar{\mathbf{A}}_1^{\mathrm{T}} \hat{\mathbf{f}} + \bar{\mathbf{B}}_1^{\mathrm{T}} \hat{\mathbf{b}} \big) \\
&+ \frac{1}{2\pi \mathrm{i}} \mathbf{B}_1 \langle \ln(z - \hat{z}_\alpha) \rangle \big(\mathbf{A}_1^{\mathrm{T}} \hat{\mathbf{f}} + \mathbf{B}_1^{\mathrm{T}} \hat{\mathbf{b}} \big), \\
&\mathrm{Im}\{z\} \geqslant 0.
\end{aligned} \tag{41}$$

Let $t = 0$ in Eqs. (40) and (41), we have

$$\mathbf{g}(z, 0) = \frac{1}{2\pi \mathrm{i}} (\bar{\mathbf{M}}_1 + \mathbf{M}_2)^{-1} (\mathbf{M}_1 + \bar{\mathbf{M}}_1) \mathbf{B}_1 \langle \ln(z - \hat{z}_\alpha) \rangle \big(\mathbf{A}_1^{\mathrm{T}} \hat{\mathbf{f}} + \mathbf{B}_1^{\mathrm{T}} \hat{\mathbf{b}} \big),$$

$$\mathbf{h}(z, 0) = \frac{1}{2\pi \mathrm{i}} (\mathbf{M}_1 + \bar{\mathbf{M}}_2)^{-1} (\bar{\mathbf{M}}_2 - \bar{\mathbf{M}}_1) \bar{\mathbf{B}}_1 \langle \ln(z - \bar{\hat{z}}_\alpha) \rangle \big(\bar{\mathbf{A}}_1^{\mathrm{T}} \hat{\mathbf{f}} + \bar{\mathbf{B}}_1^{\mathrm{T}} \hat{\mathbf{b}} \big) + \frac{1}{2\pi \mathrm{i}} \mathbf{B}_1 \langle \ln(z - \hat{z}_\alpha) \rangle \big(\mathbf{A}_1^{\mathrm{T}} \hat{\mathbf{f}} + \mathbf{B}_1^{\mathrm{T}} \hat{\mathbf{b}} \big), \tag{42}$$

which are the results for a line force and line dislocation interacting with a perfect bimaterial interface (Ting, 1992). On the other hand if we let $t = \infty$ in Eqs. (40) and (41), we arrive at

$$\mathbf{g}(z, \infty) = \frac{1}{2\pi \mathrm{i}} \begin{bmatrix} 0 & 0 & 0 \\ \frac{M_{12}^{(1)} + \bar{M}_{12}^{(1)}}{M_{22}^{(1)} + M_{22}^{(2)}} & \frac{2M_{22}^{(1)}}{M_{22}^{(1)} + M_{22}^{(2)}} & \frac{M_{23}^{(1)} + \bar{M}_{23}^{(1)}}{M_{22}^{(1)} + M_{22}^{(2)}} \\ 0 & 0 & 0 \end{bmatrix} \mathbf{B}_1 \langle \ln(z - \hat{z}_\alpha) \rangle \big(\mathbf{A}_1^{\mathrm{T}} \hat{\mathbf{f}} + \mathbf{B}_1^{\mathrm{T}} \hat{\mathbf{b}} \big),$$

$$\begin{aligned}
\mathbf{h}(z, \infty) = {}& \frac{1}{2\pi \mathrm{i}} \begin{bmatrix} 1 & 0 & 0 \\ -\frac{M_{12}^{(1)} + \bar{M}_{12}^{(1)}}{M_{22}^{(1)} + M_{22}^{(2)}} & \frac{M_{22}^{(2)} - M_{22}^{(1)}}{M_{22}^{(1)} + M_{22}^{(2)}} & -\frac{M_{23}^{(1)} + \bar{M}_{23}^{(1)}}{M_{22}^{(1)} + M_{22}^{(2)}} \\ 0 & 0 & 1 \end{bmatrix} \bar{\mathbf{B}}_1 \langle \ln(z - \bar{\hat{z}}_\alpha) \rangle \big(\bar{\mathbf{A}}_1^{\mathrm{T}} \hat{\mathbf{f}} + \bar{\mathbf{B}}_1^{\mathrm{T}} \hat{\mathbf{b}} \big) \\
&+ \frac{1}{2\pi \mathrm{i}} \mathbf{B}_1 \langle \ln(z - \hat{z}_\alpha) \rangle \big(\mathbf{A}_1^{\mathrm{T}} \hat{\mathbf{f}} + \mathbf{B}_1^{\mathrm{T}} \hat{\mathbf{b}} \big),
\end{aligned} \tag{43}$$

which are the results for a line force and a line dislocation interacting with a free-sliding interface. With the solutions for $\mathbf{h}(z, t)$ and $\mathbf{g}(z, t)$, one can then easily find the full-field solutions for the two original analytic functions $\mathbf{f}_1(z, t)$ in the upper half-plane and $\mathbf{f}_2(z, t)$ in the lower half-plane as

$$\begin{aligned}
\mathbf{f}_2(z, t) = {}& \frac{1}{2\pi \mathrm{i}} \sum_{k=1}^{3} \langle \ln(z_\alpha^* - \hat{z}_k) \rangle \mathbf{B}_2^{-1} \begin{bmatrix} 0 & 0 & 0 \\ \frac{M_{12}^{(1)} + \bar{M}_{12}^{(1)}}{M_{22}^{(1)} + M_{22}^{(2)}} & \frac{2M_{22}^{(1)}}{M_{22}^{(1)} + M_{22}^{(2)}} & \frac{M_{23}^{(1)} + \bar{M}_{23}^{(1)}}{M_{22}^{(1)} + M_{22}^{(2)}} \\ 0 & 0 & 0 \end{bmatrix} \mathbf{B}_1 \mathbf{I}_k \big(\mathbf{A}_1^{\mathrm{T}} \hat{\mathbf{f}} + \mathbf{B}_1^{\mathrm{T}} \hat{\mathbf{b}} \big) \\
&+ \frac{1}{2\pi \mathrm{i}} \sum_{k=1}^{3} \sum_{m=1}^{2} \langle \ln(z_\alpha^* - \hat{z}_k - \mathrm{i}t/\lambda_m) \rangle \mathbf{B}_2^{-1} \begin{bmatrix} 1 & 0 \\ -\frac{M_{12}^{(1)} + \bar{M}_{12}^{(2)}}{M_{22}^{(1)} + M_{22}^{(2)}} & -\frac{\bar{M}_{23}^{(1)} + M_{23}^{(2)}}{M_{22}^{(1)} + M_{22}^{(2)}} \\ 0 & 1 \end{bmatrix} \\
&\times \Omega \tilde{\mathbf{I}}_m \boldsymbol{\Omega}^{-1} \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{J}_3 \end{bmatrix} (\bar{\mathbf{M}}_1 + \mathbf{M}_2)^{-1} (\mathbf{M}_1 + \bar{\mathbf{M}}_1) \mathbf{B}_1 \mathbf{I}_k \big(\mathbf{A}_1^{\mathrm{T}} \hat{\mathbf{f}} + \mathbf{B}_1^{\mathrm{T}} \hat{\mathbf{b}} \big),
\end{aligned} \tag{44}$$

$$\begin{aligned}
\mathbf{f}_1(z, t) = {}& \frac{1}{2\pi \mathrm{i}} \sum_{k=1}^{3} \langle \ln(z_\alpha - \bar{\hat{z}}_k) \rangle \mathbf{B}_1^{-1} \begin{bmatrix} 1 & 0 & 0 \\ -\frac{M_{12}^{(1)} + \bar{M}_{12}^{(1)}}{M_{22}^{(1)} + M_{22}^{(2)}} & \frac{M_{22}^{(2)} - M_{22}^{(1)}}{M_{22}^{(1)} + M_{22}^{(2)}} & -\frac{M_{23}^{(1)} + \bar{M}_{23}^{(1)}}{M_{22}^{(1)} + M_{22}^{(2)}} \\ 0 & 0 & 1 \end{bmatrix} \bar{\mathbf{B}}_1 \mathbf{I}_k \big(\bar{\mathbf{A}}_1^{\mathrm{T}} \hat{\mathbf{f}} + \bar{\mathbf{B}}_1^{\mathrm{T}} \hat{\mathbf{b}} \big) \\
&+ \frac{1}{2\pi \mathrm{i}} \sum_{k=1}^{3} \sum_{m=1}^{2} \langle \ln(z_\alpha - \bar{\hat{z}}_k + \mathrm{i}t/\lambda_m) \rangle \mathbf{B}_1^{-1} \begin{bmatrix} -1 & 0 \\ \frac{\bar{M}_{12}^{(1)} + M_{12}^{(2)}}{M_{22}^{(1)} + M_{22}^{(2)}} & \frac{M_{23}^{(1)} + \bar{M}_{23}^{(2)}}{M_{22}^{(1)} + M_{22}^{(2)}} \\ 0 & -1 \end{bmatrix}
\end{aligned}$$

$$\times \bar{\varOmega}\tilde{\mathbf{I}}_m\bar{\boldsymbol{\Omega}}^{-1}\begin{bmatrix}\mathbf{J}_1\\ \mathbf{J}_3\end{bmatrix}(\mathbf{M}_1+\bar{\mathbf{M}}_2)^{-1}(\mathbf{M}_1+\bar{\mathbf{M}}_1)\bar{\mathbf{B}}_1\mathbf{I}_k\big(\bar{\mathbf{A}}_1^{\mathrm{T}}\hat{\mathbf{f}}+\bar{\mathbf{B}}_1^{\mathrm{T}}\hat{\mathbf{b}}\big)$$
$$+\frac{1}{2\pi\mathrm{i}}\big\langle\ln(z_\alpha-\hat{z}_\alpha)\big\rangle\big(\mathbf{A}_1^{\mathrm{T}}\hat{\mathbf{f}}+\mathbf{B}_1^{\mathrm{T}}\hat{\mathbf{b}}\big), \tag{45}$$

where

$$\tilde{\mathbf{I}}_1=\mathrm{diag}[1 \quad 0], \qquad \tilde{\mathbf{I}}_2=\mathrm{diag}[0 \quad 1]. \tag{46}$$

The time-dependent displacement and stress fields induced by the line force and line dislocation can be obtained by substituting Eqs. (44) and (45) into Eq. (6).

4. Conclusions

We have derived the time-dependent 2D Green's functions for an anisotropic bimaterial with a viscous interface due to a line force and line dislocation by means of the Stroh formalism. The results derived in this research can be further used to investigate: (1) a polygonal inclusion with uniform eigenstrains interacting with the viscous bimaterial interface; (2) a crack interacting with the viscous bimaterial interface.

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